

Variational study of the one dimensional $t - J$ model

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We find the Gutzwiller projected Fermi sea wave function(GWF) has the correct phase structure to describe the kink nature of the doped holes in the ground state of the one dimensional $t - J$ model. We find the failure of the GWF for general value of J/t and electron density n can be attributed to the residual charge correlation in the ground state. We find such residual charge correlation is well described by a XXZ-type effective Hamiltonian. Based on these observations, a Pfaffian-type variational wave function is proposed and is found to reproduce correctly the global phase diagram and corresponding correlation functions of the one dimensional $t - J$ model, including the Luther-Emery phase in the low electron density and large J/t region.

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I. INTRODUCTION

The Gutzwiller projected wave functions(GWFs) are widely used to approximate the ground state of the $t - J$ model and the Heisenberg model. In these models, local electronic correlation, as manifested in the no double occupancy constraint of electrons, plays a vital role in determining the low energy physics. Such strong local correlation make these systems difficult to study analytically. In the variational approach based on GWFs, these models are first treated in the mean field approximation in which the local constraint is relaxed to a global one. Then the local constraint is enforced afterwards by the Gutzwiller projection which simply filters out the unphysical components with doubly occupied sites in the mean field state.

The above variational strategy is used extensively in the study of the high temperature superconductors and quantum antiferromagnets. After many years of efforts, it is now believed that the Gutzwiller projected d-wave BCS state describe well the superconducting state of the high temperature superconductors[1, 2, 3, 4]. Quite recently, progress is also made on the understanding of the quasiparticle properties above such a state[5, 6, 7, 8]. The same kind of wave function is also used in recent studies on the exotic orders and exotic excitations of frustrated quantum antiferromagnet[9, 10, 11, 12].

An unresolved issue about the GWF is that it is not clear if the posteriorly executed projection can capture the kinematic effect of the local constraints, even qualitatively. In this paper, we address this issue with the one dimensional $t - J$ model.

The one dimensional $t - J$ model has been studied extensively by a broad band of methods including Bethe-Ansatz solution, conformal field theories[13, 14, 15], quantum Monte carlo[16], exact diagonalization[17], and also Variational Monte Carlo calculations[18, 19, 20, 21,

22, 23, 24]. Many properties concerning the ground state of this model are now well established. This give us the unique opportunity to judge the validity of a given approximation. The one dimensional $t - J$ model is exactly soluble at $J/t = 0$ and $J/t = 2$. For $J/t = 0$, the spin and the charge degree of freedom of the system are totally separated[18]. The spin part is described by the Heisenberg model on the squeezed chain with doped holes removed, while the charge part is described by a noninteracting spinless Fermion system. For $J/t = 2$, the system is supersymmetric and it is found that the GWF provides a fairly accurate approximation for the ground state of the system[15, 22]. For general value of J/t and electron density n , the system is a Tomonaga-Luttinger liquid(TLL) below a critical value J_c/t around 2.5. The correlation exponent of the TLL varies continuously with J/t and n [17]. For $J/t > J_c/t$, the system is unstable toward phase separation. For small n and $J/t > 2$, there is also a small region in which the system exhibit a spin gap[21, 24].

The Gutzwiller projected Fermi sea wave function and its variants has long been used to describe the ground state of the one dimensional $t - J$ model. It is well known that this wave function provides an excellent description of the undoped case of the model, namely the spin $\frac{1}{2}$ Heisenberg spin chain[19]. However, the same wave function is not that satisfactory for the doped system, except for the supersymmetric case of $J/t = 2$. For example, it fails to predict the TLL behavior in the small J/t region. A $2k_F$ peak in the spin structure factor is also missed by this wave function. Since the wave function is parameter free, it also gives no clue on the origin of the spin gap state and the phase separation at large J/t .

It is generally believed that the problems with the GWF originate from the insufficient account of the charge correlation in the system. Along this line of thinking, various kind of Jastrow factor are proposed to rem-

edy the drawbacks of GWF. For example, Hellberg and Mele introduced a long range Jastrow factor of the form $|F(r_{i\uparrow}, r_{j\downarrow})|^\nu$ and succeed in reproducing the TLL behavior, where $|F(r_{i\uparrow}, r_{j\downarrow})|^\nu$ is a Slater determinant of all the electron positions[20]. Yokoyama and Ogata found a short range repulsive Jastrow factor is able to restore the $2k_F$ peak in the spin structure factor, while a sufficiently attractive Jastrow factor can cause phase separation[22]. However, both wave functions have difficulties in reproducing the correct phase diagram. For example, the spin gap state is missed in both wave functions. At the same time, both wave functions predict a fully phase-separated state along the boundary of phase separation, which is in fact an oversimplification[17]. More importantly, no understanding on the physical origin of the proposed Jastrow factor is available and it is hard to judge if a similar modification is relevant for higher dimensional system.

For the sake of possible extension to higher dimensional case, it is important to know the reason that the simple GWF fails before any modification on it is made. As mentioned above, it is the residual charge correlation in the system which is responsible for the failure of GWF. In this paper we make this statement more precise by showing that the GWF has the correct phase structure to describe the kink nature of the doped holes in the ground state of the one dimensional $t - J$ model. In fact, we find the spin structure factor of the GWF in the squeezed chain coordinate is almost identical to that of a half filled spin chain. Thus the missing $2k_F$ peak in the spin structure factor for small J/t should be recovered if the removed holes are reinserted into the squeezed chain in the right manner.

The physical origin of the residual charge correlation can be easily seen if one reformulate the GWF in terms of the slave Boson theory[25, 26]. In the slave Boson theory, the constrained electron operator is decomposed as $\hat{c}_{i,\sigma}^\dagger = f_{i,\sigma}^\dagger b_i$, in which $f_{i,\sigma}^\dagger$ is a spin $\frac{1}{2}$ neutral Fermion called spinon and b_i is a spinless charge 1 Boson called holon. The local constraint now takes the form of an equality, $\sum_\sigma f_{i,\sigma}^\dagger f_{i,\sigma} + b_i^\dagger b_i = 1$. In terms of the slave Boson theory, the GWF corresponds to a state with all holon condensed into the zero momentum state. However, the holon is not a true Boson but a hard core Boson as a result of the local constraint. For general value of J/t , there is also an effective attraction between the holons caused by the exchange term of the $t - J$ model. Thus a XXZ-type effective Hamiltonian should be a good approximation for the residual charge correlation.

Based on these observations, a Pfaffian-type variational wave function is proposed for the ground state of the one dimensional $t - J$ model. This wave function, which has only one parameter, reproduce well the global phase diagram of the model, including the Luther-Emery(LE) phase in the small n and large J/t region. It is found that this wave function also reproduces well var-

ious correlation functions of the system and provides a refined picture for the phase separation at large J/t .

The paper is organized as follows. Section II is devoted to the investigation of the properties of the GWF. In Section III, the new variational scheme and the Pfaffian-type wave function are introduced. The phase diagram and correlation functions determined from this new variational wave function are presented in Section IV. Section V summarize the paper and includes a discussion on related issues.

II. THE GWF

The one dimensional $t - J$ model reads

$$\mathcal{H} = -t \sum_{i,\sigma} (\hat{c}_{i\sigma}^\dagger \hat{c}_{i+1,\sigma} + h.c.) + J \sum_i (\mathbf{S}_i \cdot \mathbf{S}_{i+1} - \frac{1}{4} n_i n_{i+1}), \quad (1)$$

in which $\mathbf{S}_i = \frac{1}{2} \sum_{\alpha\beta} \hat{c}_{i\alpha}^\dagger \sigma_{\alpha\beta} \hat{c}_{i\beta}$ and $n_i = \sum_\alpha \hat{c}_{i\alpha}^\dagger \hat{c}_{i\alpha}$. The electron in this model is subjected to the constraint of no double occupancy

$$\sum_\alpha \hat{c}_{i\alpha}^\dagger \hat{c}_{i\alpha} \leq 1. \quad (2)$$

The ground state of the one dimensional $t - J$ model is governed by a well defined phase structure. This can be most easily seen at half filling when the system reduces to the Heisenberg spin chain. For the Heisenberg model, it is well known that the ground state satisfy the Marshall sign rule[27, 28]. The rule says that the ground state wave function is real in the Ising basis and its sign is given by $(-1)^{N_\downarrow}$ up to a global phase, where N_\downarrow denotes the number of down spins in the even sublattice. This sign rule is a manifestation of the antiferromagnetic spin correlation in the ground state. With such a sign rule, one easily verify that $\langle \mathbf{S}_i \cdot \mathbf{S}_j \rangle \leq 0$ for i and j belonging to different sublattices and $\langle \mathbf{S}_i \cdot \mathbf{S}_j \rangle \geq 0$ for i and j belonging to the same sublattice.

The ground state at finite doping is governed by a similar sign rule. It can be easily checked that all matrix elements of the $t - J$ Hamiltonian are negative definite in a wave function that satisfy the Marshall sign rule on the squeezed chain. The squeezed chain is the chain in which the sites occupied by the doped holes are removed. This can be seen by noting that the motion of holes in this model do not disturb the spin configuration on the squeezed chain. Thus, the ground state of the one dimensional $t - J$ model should satisfy such a modified Marshall sign rule. With such a modified Marshall sign rule, one easily see that the holes in the ground state behaves as an antiphase domain wall for spin.

Now we show that the GWF satisfies the Marshall sign rule on the squeezed chain. The GWF reads

$$|\text{GWF}\rangle = \prod_i (1 - n_{i\uparrow} n_{i\downarrow}) |\text{FS}\rangle, \quad (3)$$

in which $|\text{FS}\rangle$ denotes the simple Fermi sea. In the natural basis $\prod_{i,j} c_{i\uparrow}^\dagger c_{j\downarrow}^\dagger |0\rangle$, the amplitude of GWF is given by the following Vandermonet determinant

$$\Psi(\{i\}, \{j\}) = \psi_{PW} \prod_{\alpha < \beta} (Z_{i_\alpha} - Z_{i_\beta}) \prod_{l < m} (Z_{j_l} - Z_{j_m}), \quad (4)$$

in which $\psi_{PW} = \exp[-ik_F(\sum_\alpha i_\alpha + \sum_l j_l)]$ is a plane wave factor, $Z_{i_\alpha} = \exp(i\frac{2\pi i_\alpha}{N})$ and $Z_{j_l} = \exp(i\frac{2\pi j_l}{N})$ are chord coordinates of up spins and down spins.

Now we exchange the up spin at site i_1 and the down spin at site j_1 . The resultant change in phase is given by

$$\Delta\Phi = \arg\left(\prod_{\alpha > 1} \frac{Z_{i_\alpha} - Z_{j_1}}{Z_{i_\alpha} - Z_{i_1}} \prod_{l > 1} \frac{Z_{j_l} - Z_{i_1}}{Z_{j_l} - Z_{j_1}}\right). \quad (5)$$

Since $|Z| = 1$, $\arg(\frac{Z_{i_\alpha} - Z_{j_1}}{Z_{i_\alpha} - Z_{i_1}})$ is nothing but the angle in the segment $Z_{i_1} - Z_{j_1}$ in the unit circle. Noting the fact that in a circle the angles in the same segment equal one another and sum of the opposite angles of quadrilaterals equals π , one easily find that $\Delta\Phi = N_c\pi$, in which N_c denotes the number of electrons between site i_1 and site j_1 . Taking into account the sign due to Fermion exchange, one find the change in phase is in accordance with the modified Marshall sign rule. Following essentially the same steps, one can also verify the case of exchanging a hole and an electron.

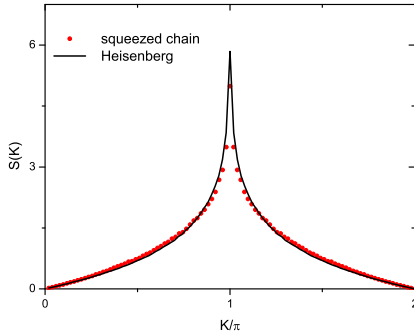


FIG. 1: Spin structure factor of the GWF in the squeezed coordinate as compared with that of a half filled spin chain. The calculation is done on a 204 sites lattice which is quarter filled.

Thus the GWF has the right phase structure to describe the ground state of the one dimensional $t-J$ model and the kink nature of the doped holes in it. In fact, this conclusion can be made even stronger. In Figure 1, we plot the spin structure factor of the GWF in the squeezed coordinate and compare it with that of a half filled spin chain. We see the two are almost identical with each other. Since the spin degree of freedom is described exactly by a Heisenberg model on the squeezed chain at $J/t = 0$, while the GWF provides an exceedingly good

approximation for $J/t = 2$, it is natural to expect that the same behavior to hold for arbitrary J/t and n .

Two conclusions follow directly from the above reasoning. First, since the spin correlation on the squeezed chain is already well described by the GWF, the missing $2k_F$ peak in the spin structure factor should be recovered if the removed holes are correctly reinserted into the squeezed chain, or, the the missing $2k_F$ peak should be attributed to the residual charge correlation in the system. Second, since the squeezed spin chain picture is argued to hold for arbitrary J/t and n , a single wave function may suffice to describe the whole phase diagram of the one dimensional $t - J$ model, including the spin gap phase at small n and large J/t .

III. THE NEW VARIATIONAL SCHEME

The origin of the residual charge correlation can be most easily seen by reformulating the GWF in terms of the slave Boson theory. In the slave Boson theory, the constrained electron operator is decomposed as $\hat{c}_{i,\sigma}^\dagger = f_{i,\sigma}^\dagger b_i$, in which $f_{i,\sigma}^\dagger$ represents the Fermionic spinon and b_i represents the Bosonic holon. In terms of these slave particles, the $t - J$ model reads

$$\mathcal{H} = \mathcal{H}_t + \mathcal{H}_J$$

$$\mathcal{H}_t = -t \sum_{i,\sigma} (f_{i,\sigma}^\dagger f_{i+1,\sigma} b_{i+1}^\dagger b_i + h.c.)$$

$$\mathcal{H}_J = \frac{J}{2} \sum_i b_i b_i^\dagger b_{i+1} b_{i+1}^\dagger (\mathbf{S}_i^f \cdot \mathbf{S}_{i+1}^f - \frac{1}{4} n_i^f n_{i+1}^f),$$

in which $\mathbf{S}_i^f = \frac{1}{2} \sum_{\alpha\beta} f_{i\alpha}^\dagger \sigma_{\alpha\beta} f_{i\beta}$ and $n_i^f = \sum_\alpha f_{i\alpha}^\dagger f_{i\alpha}$. The no double occupancy constraint now takes the form of an equality

$$\sum_\alpha f_{i\alpha}^\dagger f_{i\alpha} + b_i^\dagger b_i = 1. \quad (6)$$

When the local constraints Eq.(6) is exactly satisfied, the factor $b_i b_i^\dagger b_{i+1} b_{i+1}^\dagger$ appearing in \mathcal{H}_J plays no role and can be neglected.

In the mean field treatment, an RVB order parameter $\chi = \sum_\alpha \langle f_{i+1\alpha}^\dagger f_{i\alpha} \rangle$ is introduced to decompose the interaction term. At the same time, the local constraint is relaxed to a global one. The mean field Hamiltonian for the spinon and the holon part read[26]

$$\mathcal{H}^f = -(tx + \frac{3J\chi}{8}) \sum_{i\sigma} (f_{i,\sigma}^\dagger f_{i+1\sigma} + h.c.)$$

$$\mathcal{H}^b = -t\chi \sum_i (b_i^\dagger b_{i+1} + h.c.),$$

in which x is the hole density. The mean field ground state is given by the product of the spinon Fermi sea and

the holon Bose condensate

$$|\Phi\rangle = (b_{k=0}^\dagger)^{N_h} \prod_{k \leq k_F} f_{k\uparrow}^\dagger f_{k\downarrow}^\dagger |0\rangle. \quad (7)$$

When this state is projected into the subspace that satisfy the constraint Eq.(6), we get the GWF.

In the mean field theory, the holon is a free Boson and condenses in the ground state. However, due to the local constraint, the holon is actually a hard core Boson which can not condense in one spatial dimension. The uncondensed nature of the hard core Boson in 1d originates from the kinematic effect of the local constraint. Due to this constraint, the Hilbert space for the one dimensional hard core Boson system becomes disconnected at the single particle level. We note for comparison that the Hilbert space of the spinon part is still connected even when the local constraint is enforced. Thus the holon should be treated as hard core Boson rather than free Boson.

Another source of the residual charge correlation is provided by the superexchange term of the $t - J$ model. When two electrons are next to each other, they enjoy an attraction due to the superexchange. This attraction is not captured by the mean field order parameter χ and should be reintroduced.

Combining these considerations, the residual charge correlation beyond the GWF should be described by the following XXZ-type effective Hamiltonian

$$\mathcal{H}_v = - \sum_i (\hat{b}_i^\dagger \hat{b}_{i+1} + h.c.) - v \sum_i \hat{b}_i^\dagger \hat{b}_{i+1}^\dagger \hat{b}_{i+1} \hat{b}_i, \quad (8)$$

in which \hat{b}_i^\dagger is the operator for hard core Boson and v is the rescaled attraction. If we denote the ground state of \mathcal{H}_v as Λ_v , then $P_G \Lambda_v |GWF\rangle$ should be a good variational wave function for the one dimensional $t - J$ model.

Although \mathcal{H}_v is exactly soluble[29], an explicit form for Λ_v is available only in limited cases. For $v = 0$, Λ_v is nothing but the Hellberg-Mele Jastrow factor with $\nu = 1$. For $v = 1$, Λ_v is a constant and our proposed wave function reduce to the GWF. At the quarter filling, \mathcal{H}_v exhibit particle-hole symmetry. In a separated paper[30], we show a Hellberg-Mele-type variational wave function provides an exceedingly good description for the ground state of the XXZ model in the $S^z = 0$ sector. However, away from the particle-hole symmetric point, the Hellberg-Mele wave function cease to be a good approximation.

For general value of v and Boson density, we have to resort to approximation. Through the Jordan-Wigner transformation, the XXZ Hamiltonian can be rewritten as

$$\mathcal{H}_v = - \sum_i (c_i^\dagger c_{i+1} + h.c.) - v \sum_i c_i^\dagger c_{i+1}^\dagger c_{i+1} c_i, \quad (9)$$

in which c_i^\dagger is a spinless Fermion. For this Hamiltonian, we adopt the BCS approximation to decouple the interaction term. The BCS ground state for the spinless Fermion reads

$$\prod_{k>0} (u_k + v_k c_k^\dagger c_{-k}^\dagger) |0\rangle, \quad (10)$$

in which $\frac{v_k}{u_k} = \frac{\Delta_k}{\epsilon_k + E_k}$, $\Delta_k = \Delta \sin(k)$, $\epsilon_k = -2 \cos(k) - \mu$ and $E_k = \sqrt{\epsilon_k^2 + \Delta_k^2}$. Here Δ is the BCS gap for the spinless Fermion and is treated as the only variational parameter in our theory (the chemical potential μ can be determined by the density equation and is not an independent parameter). In real space, the BCS state for the spinless Fermion takes the form of a Pfaffian. A Pfaffian is a square root of the determinant of a antisymmetric matrix of even order[31]. In our case, the matrix element of the antisymmetric matrix is given by

$$f_{i,j} = \sum_{k>0} \frac{v_k}{u_k} \sin(k(i-j)), \quad (11)$$

in which i and j denote the coordinates of the spinless Fermions. Thus our variational wave function for the one dimensional $t - J$ model is given by

$$\Psi = \text{Pf}(\Delta) |GWF\rangle, \quad (12)$$

in which $\text{Pf}(\Delta)$ is the Pfaffian for the holes which are now spinless Fermions.

IV. RESULTS

A. Ground state phase diagram

The ground state phase diagram determined from the Pfaffian-type wave function is presented in Figure 2.

The phase diagram contains three distinct phases. For small and intermediate value of J/t , the system is in the TLL phase in which both charge and spin excitation are gapless. For larger value of J/t , the system is unstable towards phase separation. At small n and large J/t , there is a small region in which the system exhibits a spin gap. In the spin gap phase, the charge excitation is still gapless. Following the convention, this phase is termed Luther-Emery liquid.

The phase boundaries are determined as follows. To illustrate the idea, we plot the variational energy as a function of the electron density for $J/t = 2$ and $J/t = 3$ in Figure 3. For $J/t = 2$, the energy curve is concave everywhere so that a homogenous phase is globally stable for all electron density. For $J/t = 3$, a convex region appears at intermediate values of electron density in the energy curve. In this case, the boundaries for the globally stable phases are given by the two tangency points shown in the figure, while the boundaries for the locally stable phases are given by the two inflexion points.

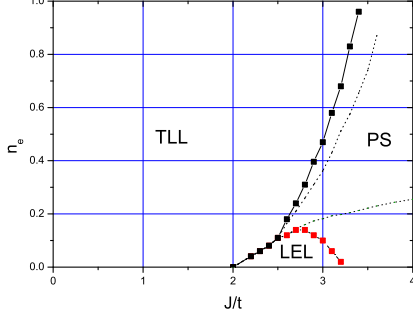


FIG. 2: Ground state phase diagram of the one dimensional $t - J$ model determined from the Pfaffian-type variational wave function. The dotted lines indicate the boundaries for the existence of locally stable phases. Here TLL denotes Tomonaga-Luttinger liquid, LEL denotes Luther-Emery liquid, while PS denotes phase separated state.

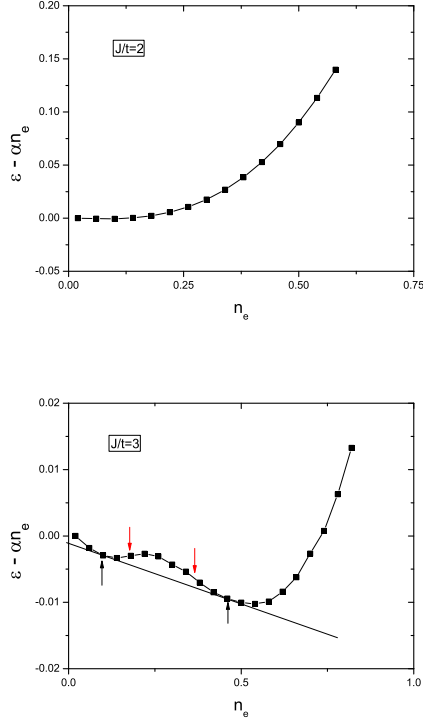


FIG. 3: Variational energy per site ϵ as a function of the electron density n_e for $J/t = 2$ and $J/t = 3$. For clarity's sake, a linear decreasing background of the energy is subtracted. $\alpha = \frac{d\epsilon}{dn_e}|_{n_e \rightarrow 0}$ is the initial slope of the energy curve. The arrows above the curve indicate the locations of the inflexion points, while the arrows below the curve indicate the locations of the tangency points. The phase boundaries are determined from these points as explained in the text.

For electron density that lies between the two tangency

points, the system is unstable towards phase separation. The density of the phase separated phases are given by two tangency points. For $2.5 < J/t < 3.2$, the system phase separates into a hole rich phase and an electron rich phase. For $3.2 < J/t < 3.4$, the hole rich phase is replaced by a empty phase. For $J/t > 3.4$, a fully phase separated state is realized in which the electron rich phase is replaced by a half filled spin chain.

The convex region of the energy curve diminishes to zero at about $J/t = 2.5$. The phase boundary between the TLL phase and the LEL phase for $J/t < 2.5$ is determined by examining the infrared behavior of the spin structure factor $S(q)$. In the spin gap phase, $S(q)$ should be quadratic at small q , while in the TLL phase a linear behavior is expected[32]. For the charge excitation, a similar criteria exists on the density structure factor $N(q)$.

The existence of the LEL phase is quite unexpected from the point of view of the mean field theory. In the mean field theory, the spinon is still described by a filled Fermi sea which is by definition gapless. However, after Gutzwiller projection the spinon get entangled with the holon. Such an entanglement change drastically the spin correlation of the system.

B. Correlation functions

Four correlation functions are evaluated in this work. They are the momentum distribution function defined as

$$n(k) = \frac{1}{2N} \sum_{i,j,\sigma} \langle c_{i\sigma}^\dagger c_{j\sigma} \rangle e^{ik(r_i - r_j)}, \quad (13)$$

the spin structure factor defined as

$$S(k) = \frac{4}{N} \sum_{i,j} \langle S_i^z S_j^z \rangle e^{ik(r_i - r_j)}, \quad (14)$$

the charge structure factor defined as

$$C(k) = \frac{1}{N} \sum_{i,j} (\langle n_i n_j \rangle - \langle n_i \rangle \langle n_j \rangle) e^{ik(r_i - r_j)}, \quad (15)$$

and the pair correlation function defined as

$$P(k) = \frac{1}{N} \sum_{i,j} \langle \Delta_i^\dagger \Delta_j \rangle e^{ik(r_i - r_j)}, \quad (16)$$

in which Δ_i is the annihilation operator for a nearest-neighboring pair

$$\Delta_i = \frac{1}{\sqrt{2}} (c_{i\uparrow} c_{i+1\downarrow} - c_{i\downarrow} c_{i+1\uparrow}). \quad (17)$$

First we present the result for the TLL phase. The correlation functions for $J/t = 0, 1$ and 2 at quarter filling are shown in Figure 4. For comparison's sake, we

also plot the result calculated from the Hellberg-Mele wave function. From the figure we see that the correlation functions calculated from the Pfaffian-type wave function are almost identical with that calculated from the Hellberg-Mele wave function, apart from the small deviations due to critical fluctuations. Since the Pfaffian is derived from a BCS mean field approximation in which a gap opens up, the residual charge correlation described by it is short ranged. Thus the Pfaffian-type wave function should exhibit Fermi-liquid behavior, as is clear in Figure 4. To recover the critical fluctuations, one should go beyond the mean field approximation.

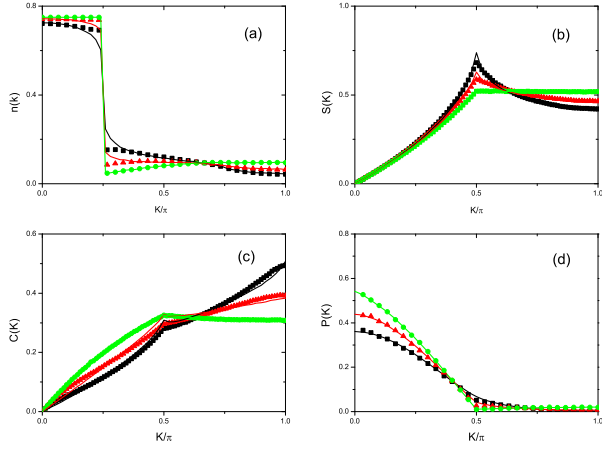


FIG. 4: (a) The momentum distribution function $n(k)$, (b) the spin structure factor $S(k)$, (c) the charge structure factor $C(k)$, and (d) the singlet pairing correlation function $P(k)$ at quarter filling for $J/t=0$ (black square), $J/t=1$ (red up triangle), and $J/t=2$ (green circle). The solid lines denote the result calculated from the Hellberg-Mele variational wave function.

For $J/t > 2$, the Pfaffian-type wave function becomes less satisfactory for the quarter filled system. In Figure 5, we plot the correlation functions of the quarter filled system at $J/t = 2.5$ and 3, the latter of which is very close to the boundary of phase separation. Near the boundary of phase separation, the Hellberg-Mele wave function starts to develop charge instability, as is clear from Fig. 5(c). This tendency is missed by the Pfaffian-type wave function. Instead, a structure at $2k_F$ remains evident in the correlation functions. This is to be expected, since we start from a Fermionic description of the residual charge correlation. In fact, it is quite amazing that the BCS approximation remains to be a good approximation for J/t as high as 2.5 (the optimized value for the BCS gap Δ is approximately given by J/t at quarter filling).

To quantify the above discussion, we plot in Figure 6 the relative error in the variational energy for both the Pfaffian-type wave function and the Hellberg-Mele wave function. For small J/t , the energy of the Pfaffian-type

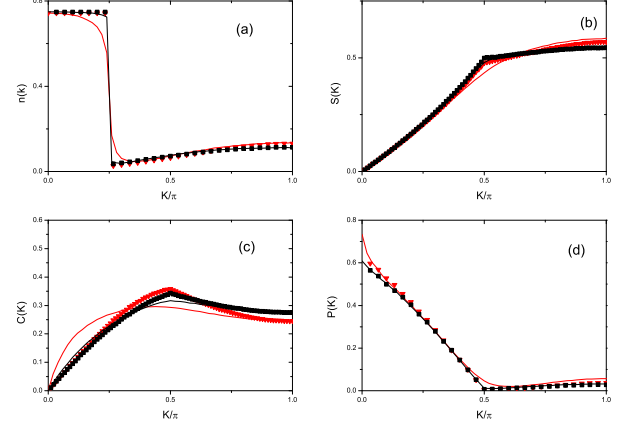


FIG. 5: (a) The momentum distribution function $n(k)$, (b) the spin structure factor $S(k)$, (c) the charge structure factor $C(k)$, and (d) the singlet pairing correlation function $P(k)$ at quarter filling for $J/t=2.5$ (black square) and $J/t=3$ (red up triangle). The solid lines denote the result calculated from the Hellberg-Mele variational wave function.

wave function is slightly lower than that of the Hellberg-Mele wave function. For larger value of J/t , the ordering is reversed. However, both wave functions give good estimate for the ground state energy before phase separation.

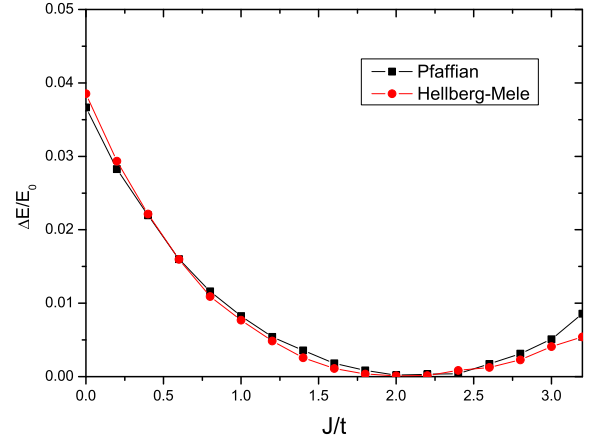


FIG. 6: Relative error in variational energy at quarter filling. The exact value of the ground state energy is taken from [22].

Although the Hellberg-Mele wave function provides a good description for the quarter filled system, it fails badly at low electron density. On the other hand, the Pfaffian-type wave function describes quite well the physics in the low density regime, including the spin gap

phase at large J/t . To illustrate this, we plot in Figure 7 the error in variational binding energy for a single pair of electrons calculated from both wave functions. From the figure we see the Pfaffian-type wave function is almost exact for all values of J/t in the low density limit. We think this explains why the spin gap phase can be correctly reproduced by the Pfaffian-type variational wave function.

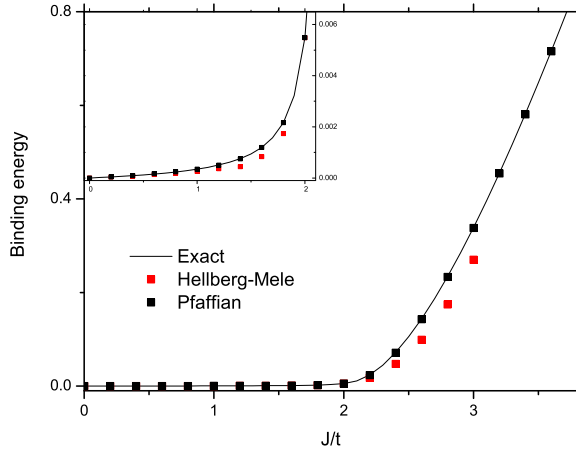


FIG. 7: Error in variational binding energy for a single pair of electrons calculated from the Pfaffian-type wave function and the Hellberg-Mele wave function. The inset shows an expanded view of the $0 < J/t < 2$ region.

Now we present the correlation functions for the LEL phase at small n and large J/t . In Figure 8, the correlation functions for $J/t = 2.8$ and $n = 0.06$, a system deeply inside the LEL phase, are plotted. As mentioned above, the spin gap manifests itself in the quadratic behavior of the spin structure factor in the small q limit. We note that spin gap state is metastable in a much larger region than that of the LEL phase.

V. SUMMARY AND DISCUSSION

In this paper, we have carried out a variational study of the one dimensional $t - J$ model. We find the failure of the simple GWF should be attributed to the residual charge correlation. Reformulating the GWF in terms of the slave Boson theory, we find the residual charge correlation should be described by a XXZ-type effective Hamiltonian. Based on this observations, a Pfaffian-type variational wave function is proposed for the one dimensional $t - J$ model. We find this wave function, which has only one variational parameter, reproduces correctly the global phase diagram and the corresponding correlation functions.

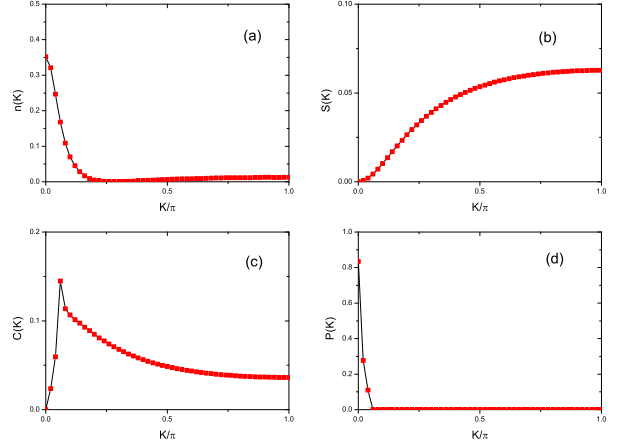


FIG. 8: Correlation functions for $J/t = 2.8$ and $n = 0.06$, a system deeply inside the spin gap phase. (a) The momentum distribution function $n(k)$, (b) the spin structure factor $S(k)$, (c) the charge structure factor $C(k)$, and (d) the singlet pairing correlation function $P(k)$.

It is interesting to note the way in which the spin correlation is affected by the charge degree of freedom in this model. Through the investigation of the phase structure of the ground state wave function, we find the doped holes behave as anti-phase domain walls for the spin correlation. We show further the spin degree of freedom of the system is well approximated by a half filled spin chain in the squeezed coordinates throughout the phase diagram. For small electron density, the effect of the charge degree of freedom on the spin part can be so drastic as to induce a gap in the excitation spectrum of the latter. This spin gap is beyond mean field description and should be attributed to the strong entanglement of spin and charge degree of freedom in the projected subspace.

It is also interesting to note the effect of the local constraint for this system. In conventional GWF, the effect of the local constraint is taken into account posteriorly by filtering out the unphysical components in the unprojected state. In this paper, we find this procedure may fail when the kinematic effect of the local constraint is essential for establishing (or, more accurately, destroying) the mean field correlation in the unprojected state. The one dimensional $t - J$ model provides a particular example of this type. In the one dimensional $t - J$ model, the Hilbert space for the charge degree of freedom is disconnected at the single particle level due to the local constraint. When the local constraint is relaxed, the connectivity of the Hilbert space for the charge degree of freedom is changed in a qualitative manner. Such a change in the connectivity of the Hilbert space is responsible for the appearance of the Bose condensation of the charged particle in the mean field theory and is ultimately

responsible for the failure of the GWF to describe the Tomonaga-Luttinger behavior of the system.

For a full understanding of the residual charge correlation in the one dimensional $t - J$ model, one should also take into account the attraction due to the exchange term. In the mean field treatment, the exchange term is decoupled in the $f_{i\sigma}^\dagger f_{j\sigma}$ channel which can not account for such a charge correlation effect. We find this attraction counteracts with the effect of the local constraint and cancels it out around $J/t = 2$. This explains the excellentness of the GWF at the supersymmetric point.

The critical behavior of the one dimensional $t - J$ model is not correctly described by the Pfaffian-type wave function. This is natural since the Pfaffian itself is derived from a BCS-type mean field treatment. Although single particle condensation is gone in this treatment, a condensate of pair of spinless Fermion still exists. To recovery the correct critical behavior, one must get rid of such a condensate. One way to achieve this is to introduce a second charge correlator of the Hellberg-Mele-type, as is done in [24] and [23]. According to our analysis, the problem of finding a good variational description for the one dimensional $t - J$ model reduces to that for the much simpler one dimensional XXZ model. We think the correct critical behavior should be recovered by a more accurate guess for the ground state wave function of the latter model.

Finally, we mention possible generalization of the idea used in this work to the study of the two dimensional $t - J$ model. In two spatial dimension, the kinematic effect of the local constraint should be less dramatic since the connectivity of the Hilbert space is not affected by the local constraint. This can also be seen from the fact that the two dimensional XXZ model does undergo Bose condensation at zero temperature. However, it is much subtler to analyze the interplay between the spin and charge degree of freedom in two spatial dimension since the two frustrate each other. Thus, the validity of the simple GWF in two dimension remains to be seen.

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